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Discussion

Mathematical statistics and computer vision ☆

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ABSTRACT

In this discussion paper, I present my views on the role on mathematical statistics for solving computer vision problems.

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When I was looking for a Ph.D. dissertation topic, I accidentally came across a paper by W. Larimore on Statistical Inference on Random Fields in the Proceedings of the IEEE [6]. This paper discussed parameter estimation and hypothesis testing methods for two-dimensional non-causal models. This paper led me to classical papers by Peter Whittle [9], Yu Rozanov [7], John Woods [10] and Julian Besag [1], as well as the paper by Besag and Moran [2] on parameter estimation in Gaussian Markov random field models. As a student in Electrical and Computer Engineering, I had been exposed to basic concepts in parameter estimation, random processes and decision theory and I was naturally attracted to the possibilities of using mathematical statistical framework for computer vision problems. I wrote my dissertation on stochastic models for image processing and understanding. Since then, I have worked on mathematical statistics-based approaches for many computer vision problems. Since most computer vision problems are concerned with *inferring* some properties (radiometric, geometric,…) from images and videos, tools from mathematical statistics are very useful solving computer vision problems. When one considers inferring 3D geometry from images and videos, computer vision problems can be immensely challenging for mathematical statisticians. Another reason, why statistical methods may be effective for computer vision problems is that one is then able to account for degradations in the data using appropriate distributions; any prior information can also be incorporated in a Bayesian framework. By throwing in non-parametric inference tools, manifolds etc., one can have even more fun.

Statistical methods were not always welcome in computer vision. In the early years, mostly linear models and Gaussian distributions were used while developing statistical inference methods for computer vision; the simplicity of these models did not find favor with leading computer vision researchers. My mentor Azriel Rosenfeld felt that unless methods that did not rely on linear models and Gaussian distributions were available, statistical methods will not scale up to the challenges of computer vision problems. Ulf Grenander was working on abstract mathematical and statistical models and methods for many computer vision problems and presented his findings in books that were not easily understood by mainstream computer vision researchers. As a result, Prof. Grenander's work was seen as esoteric. David Cooper was also vigorously pursuing Bayesian methods for boundary and object recognition [4]. A seismic shift occurred in 1984 thanks to the seminal paper on simulated annealing, stochastic relaxation and MAP restoration of images by Geman and Geman [5] that appeared in PAMI. This paper demonstrated that foundational methods from mathematical statistics can and will make a significant impact on computer vision. I believe from this moment on statistical models and methods became acceptable to the computer vision community. The paper by Geman and Geman opened a floodgate of papers on image segmentation, restoration, classification, optical flow estimation, etc. A deterministic alternative known as the iterated condition mode was presented in the paper by Besag [3] on statistical analysis of dirty pictures that appeared in the Journal of Royal Statistical Society. The simulated annealing algorithm inspired algorithms like mean field annealing, graduated non-convexity and maximum posterior marginal. The Bayesian formulation is an entrenched methodology vigorously being pursued by numerous computer vision researchers (for example, Geman, Yuille, Zhu and many others). MRFs are here to stay in computer vision. Many optimization methods that are currently popular have their roots in optimization of posterior probability density functions derived using MRF representations. A good example of the impact of MRF-driven methods is the well cited paper on the comparison of energy minimization methods for MRFs that appeared in PAMI in 2008 [8].

It is often said that timing is everything even in scholarly research pursuits. Principal Component Analysis (PCA) is a well known dimensionality reduction in statistics. When PCA was applied to face representation and recognition in the late eighties and early nineties, it generated a tsunami of papers leading to various subspace-based

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approaches for face and object recognition. Methods based on Fishers Linear Discriminant Analysis (LDA), kernel-PCA, kernel-LDA and numerous variations thereof (for example, partial least squares) have been developed over the last two decades. The application of Support Vector Machines (SVM) to problems ranging from OCR to face recognition is another positive example of the impact of methods rooted in mathematics statistics in solving computer vision problems. Although these methods appear to be effective, they do not work well with variations due to pose, illumination variations, occlusions etc. Recently, methods based on domain adaptation are being developed for designing classifiers that can adapt to the so called domain shifts due to pose, illumination, blur, etc. This appears to be a promising approach.

Another well known example that clearly illustrates the impact of statistical inference on computer vision is the emergence of particle filters. Although the idea of particle filtering was known to the radar tracking community and as jump diffusion process in stochastic filtering world, the adaptation of particle filters to a tracking problem (by Isard and Blake) that computer vision researchers can relate to contributed to its immense success. In fact, the particle filter has become one of the major tools in the computer vision tool kit, that even students who have not been exposed to random process or estimation theory or Kalman filter are able to use it with ease! The lesson to be learned here is that if a computer vision algorithm can be in OpenCV or MATLAB, it has a long life! On the flip side, it is not a good idea that folks who have not been exposed to even the basics of random processes and estimation theory should be applying particle filters. This can be discussed in a separate article on how to educate a computer vision researcher.

The last example I would like to mention is the statistical analysis of boundaries, shapes using landmarks, manifolds etc. Following the seminal works of Kendall, Mardia, Grenander, Cooper and others, recent efforts by Anuj Srivastava, Laurent Younes, Mike Miller and others have brought new models and methods based on differential geometry and statistical inference to bear fruits in an important area of computer vision. Statistical inference on manifolds for object, event, and gesture recognition is gaining acceptance by computer vision community.

Other concepts from mathematical statistics that have impacted the computer vision area are robust methods for computer vision, performance evaluation, Monte Carlo techniques, Lasso and ensemble learning. Space limitations do not permit an elaborate discussion of these topics.

Developing effective statistical inference methods that can handle structured (or geometric) data is something that should be of interest to computer vision researchers and mathematical statisticians. One of my teachers at Purdue, Prof. K.S. Fu used to say that statistical methods that cannot handle structure would not be very effective for pattern recognition and image understanding and that is why he strongly believed in designing grammars for pattern analysis and recognition problems. In the late seventies and early eighties, Prof. Fu and his students introduced stochastic grammars and associated inference methods for many image analysis and pattern recognition problems. The design and inference of stochastic grammars, ontologies, Markov logic networks, etc. for large scale computer vision problems have received much attention during the past decade and will continue to be a productive area of research. Past and ongoing research in the area of probabilistic reasoning and graphical models pioneered by Pearl and Jordan respectively provide ample inspiration for integrating structure and statistical inference.

Given the diversity of computer vision research area (computer scientists, electrical engineers, statisticians, neuroscientists, and psychophysicists are involved), it is tempting for one or more groups of researchers to claim that “their view of the elephant” is the best one. This is not productive. Over the years, I have learnt and my students have taught me how to apply methods from Statistics (GMRFs, Cramer–Rao bounds, Fisher–Rao metric, inference on manifolds), Electrical Engineering (Kalman filters, dynamic models, information theory, Cramer–Rao bounds), and Computer Science (graph matching, stochastic Petri nets, non-monotonic reasoning and sub-modular functions) for a wide variety of computer vision problems. I have enjoyed all of these and feel computer vision provides sufficient space for anyone who is earnest enough to get involved. Let us all have a pleasant ride for the years to come.

In sum, mathematical statistical methods have played and will continue to play a big role to play in the development of robust algorithms for many computer vision problems. In order to work in this area or appreciate ongoing works in this area, the students are well advised to take additional courses in statistical inference, differential geometry, random processes, estimation theory, linear system theory, and optimization techniques.

In writing this article, I may have inadvertently omitted many related works of significant impact. Space limitations do not permit elaborate discussions of all the efforts that are related to the theme of this article. My sincere apologies to those who have promoted the application of mathematical statistics principles to computer vision problems but are not mentioned here.

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